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LETTER TO THE EDITOR

Neural networks: translation-, rotation- and scale-invariant pattern recognition

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Abstract. A neural network model which is capable of recognising transformed versions of a set of learnt patterns is proposed. The group of transformations includes global translations, rotations and scale transformations. The neural firing thresholds are used as additional degrees of freedom.

Statistical models of neural networks, proposed by Hopfield (1982) and Little (1974), have been proved to function as systems of a content-addressable memory with high robustness (Amit *et al* 1985, Hopfield 1984, Hopfield *et al* 1983). It was shown that, in terms of such rather simple Ising spin systems, a variety of very promising memory models could be formulated. Among them are models of hierarachical memory developed by Parga and Virasoro (1986), Dotsenko (1985, 1986), Gutfreund (1987), layered systems developed by Domany *et al* (1986), systems with random asymmetric interactions investigated by Hertz *et al* (1986) and Feigelman and Ioffe (1987), diluted systems investigated by Derrida *et al* (1987) and Treves and Amit (1988), an elegant model with biologically motivated asymmetry of interactions invented by Shinomoto (1987), a neural system of a temporal association with ordered asymmetry of interactions developed by Sompolinsky and Kanter (1986), and many others.

All these systems are supposed: (i) to store in the memory a certain number of spin patterns $\{\xi_i^{(\alpha)}\}$ ($\xi_i = \pm 1$, $i = 1, 2, \ldots, N$, $\alpha = 1, 2, \ldots, p$) and (ii) to retrieve the correct learnt pattern from a distorted one via simple dynamical equations for spin variables. Correspondingly, the problems which are considered are: (i) the learning rules (how the parameters of the models are defined in terms of $\xi_i^{(\alpha)}$) and the 'architecture' of the models, and (ii) the process of retrieval.

The key parameters for the learning process are the overlaps:

$$q^{\alpha\beta} = \frac{1}{N} \sum_i \xi_i^{(\alpha)} \xi_i^{(\beta)} \tag{1}$$

which describe how close the patterns in the phase space are. Accordingly the effective variables

$$m^{(\alpha)}(t) = \frac{1}{N} \sum_i \xi_i^{(\alpha)} \sigma_i(t) \tag{2}$$

describe how close the current spin pattern $\{\sigma_i(t)\}$ is to the learnt patterns.

The problem which will be considered in this letter and which cannot be solved via direct minimisation of the M in the original Hopfield model is the following. Any

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two pictures which, by common sense, are identical (or close) but differ by a uniform shift (rotation, scale transformation, etc) could be treated as absolutely different in terms of overlaps (1) and (2). As far as I know there are only two papers, by von der Malsburg and Bienenstock (1987) and Kree and Zippelius (1988), in which this problem was considered. The idea of the present letter was initiated by the approach of Kree and Zippelius (1988) who extended an architecture of the Hopfield model for the recognition of topologically isomorphic classes of *graphs.* The present investigation is restricted only to translation-, rotation- and scale-invariant recognition of *patterns* of the σ and is performed almost in terms of the original Hopfield model.

The idea is to use neural firing thresholds as additional degrees of freedom and to introduce the parameters of global translations $\boldsymbol{a}$, rotations θ and scale transformations λ as additional 'slow' dynamical variables via $\sigma^{(0)}([\lambda\hat{\theta}\boldsymbol{r}]+\boldsymbol{a})$, where $\sigma^{(0)}(\boldsymbol{r})$ is the proposed pattern to be recognised and [...] denotes the whole part. Since the sites of the lattice will be assumed to form a 2D 'screen', the discrete vector $\boldsymbol{r}$ will be used instead of the subscript i.

The model should be composed in such a way that the minimisation of the variables

$$M^{(\alpha)}(\boldsymbol{a},\theta,\lambda)=\frac{1}{N}\sum_{\boldsymbol{r}}\xi^{(\alpha)}(\boldsymbol{r})\sigma^{(0)}([\lambda\hat{\theta}\boldsymbol{r}]+\boldsymbol{a}) \tag{3}$$

over $\boldsymbol{a}$, θ and λ should occur first. After that should be an ordinary retrieval process of the Hopfield model:

$$H=-\tfrac{1}{2}\sum_{\boldsymbol{rr}'}J_{\boldsymbol{rr}'}\sigma(\boldsymbol{r})\sigma(\boldsymbol{r}') \qquad J_{\boldsymbol{rr}'}=\frac{J_0}{N}\sum_{\alpha=1}^{p}\xi^{(\alpha)}(\boldsymbol{r})\xi^{(\alpha)}(\boldsymbol{r}') \tag{4}$$

where the starting values of the σ should be given by the optimal pattern $\sigma^{(0)}([\lambda^*\hat{\theta}^*\boldsymbol{r}]+\boldsymbol{a}^*)$.

It will be shown that, using the degrees of freedom of neural firing thresholds (which are external fields for spins), these two stages can be incorporated self-consistently in one model. For simplicity, only translations will be considered first. The generalisation for rotations and scaling will be straightforward.

Let the model be described by the Hamiltonian:

$$H=-\frac{1}{2}\sum_{\boldsymbol{rr}'}J_{\boldsymbol{rr}'}\sigma(\boldsymbol{r})\sigma(\boldsymbol{r}')-\sum_{\boldsymbol{r}}\varepsilon(\boldsymbol{r})\sigma(\boldsymbol{r}). \tag{5}$$

The relaxation dynamics of the Ising variables is described by the usual discrete equations:

$$\sigma(\boldsymbol{r};t+1)=\begin{cases}\sigma(\boldsymbol{r};t) & \text{with probability } p_+=\dfrac{\exp[\beta h(\boldsymbol{r},t)\sigma(\boldsymbol{r},t)]}{2\cosh[\beta h(\boldsymbol{r},t)\sigma(\boldsymbol{r},t)]}\\ -\sigma(\boldsymbol{r},t) & \text{with probability } p_-=1-p_+\end{cases} \tag{6}$$

where

$$h(\boldsymbol{r};t)=\sum_{\boldsymbol{r}'}J_{\boldsymbol{rr}'}\sigma(r';t)+\varepsilon(\boldsymbol{r};t). \tag{7}$$

The proposed pattern $\sigma^{(0)}(\boldsymbol{r})$ is supposed to be mapped onto the ε as follows:

$$\varepsilon(\boldsymbol{r};t)=h_0\sigma^{(0)}(\boldsymbol{r}+\boldsymbol{a}(t)). \tag{8}$$

It could be assumed that, for example, periodic boundary conditions are imposed.

Here $\boldsymbol{a}(t)$ is an additional dynamic variable which obeys the relaxation equation:

$$\dot{\boldsymbol{a}}(t)\equiv\boldsymbol{a}(t+1)-\boldsymbol{a}(t)=-\delta H/\delta\boldsymbol{a}+\boldsymbol{\eta}(t) \tag{9}$$

where $\boldsymbol{\eta}(t)$ is an ordinary temperature noise. It is assumed that $N^{-1} \ll h_0/J_0 \ll 1$, $h_0 \ll T$ and

$$\sigma(\boldsymbol{r};\, t=0) = \sigma^{(0)}(\boldsymbol{r}). \tag{10}$$

Equation (7) for the local field $h(\boldsymbol{r};\, t)$ can be represented as follows:

$$h(\boldsymbol{r};\, t) = J_0 \sum_{\alpha=1}^{p} \xi^{(\alpha)}(\boldsymbol{r}) M^{(\alpha)}(t) + h_0 \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}(t)). \tag{11}$$

Therefore, as long as $M^{(\alpha)}(t) \ll h_0/J_0$ the second term in (11) dominates and the spins follow the fields $\{h_0\sigma^{(0)}\}$:

$$\begin{aligned} \langle \sigma(\boldsymbol{r};\, t)\rangle &\simeq \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}(t)) \tanh \beta h_0 \\ &\simeq \beta h_0 \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}(t)). \end{aligned} \tag{12}$$

In this case the effective potential for the variable $\boldsymbol{a}$ is

$$\begin{aligned} E(\boldsymbol{a}) &\simeq -\tfrac{1}{2}\beta^2 h_0^2 \sum_{rr'} J_{rr'} \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}) \sigma^{(0)}(\boldsymbol{r}' + \boldsymbol{a}) \\ &= -\tfrac{1}{2} N \beta^2 h_0^2 J_0 \sum_{\alpha=1}^{p} (M_0^{(\alpha)}(\boldsymbol{a}))^2 \end{aligned} \tag{13}$$

where

$$M_0^{(\alpha)}(\boldsymbol{a}) = \frac{1}{N} \sum_{\boldsymbol{r}} \xi^{(\alpha)}(\boldsymbol{r}) \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}). \tag{14}$$

Therefore the search for the 'correct' pattern at this stage is a wandering over the states of the N-dimensional hypercube along the 2D 'subspace' defined by $\sigma^{(0)}(\boldsymbol{a})$. This is in contrast to the usual relaxation in the Hopfield model where the wandering could proceed in all directions on the hypercube since all the spins are independent.

It is supposed that, for some $\boldsymbol{a}^*$, the proposed pattern $\{\sigma^{(0)}\}$ has a finite overlap with one of the learnt patterns, say $\{\xi^{(\alpha_0)}\}$:

$$\frac{1}{N} \sum_{\boldsymbol{r}} \xi^{(\alpha_0)}(\boldsymbol{r}) \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{a}^*) = q_{\alpha_0} \sim \mathrm{O}(1) \tag{15}$$

and has no finite overlap with the others: $M^{(\alpha)}(\boldsymbol{a}) \sim \mathrm{O}(1/N^{1/2})$ $(\alpha \neq \alpha_0)$.

It is also important to assume that all the patterns we are dealing with have *finite spatial correlation length* R_c, i.e.

$$\begin{aligned} &\frac{1}{N} \sum_{\boldsymbol{r}} \sigma^{(0)}(\boldsymbol{r}) \sigma^{(0)}(\boldsymbol{r} + \boldsymbol{R}) \sim \exp(-|\boldsymbol{R}|/R_c) \\ &\frac{1}{N} \sum_{\boldsymbol{r}} \xi^{(\alpha)}(\boldsymbol{r}) \xi^{(\alpha)}(\boldsymbol{r} + \boldsymbol{R}) \sim \exp(-|\boldsymbol{R}|/R_c). \end{aligned} \tag{16}$$

Otherwise there will be no 'attraction' of the patterns.

The effective attraction to the correct pattern will therefore be described by the potential:

$$E_{\alpha_0}(a) \sim -\tfrac{1}{2} N \beta^2 h_0^2 J_0 q_{\alpha_0}^2 \exp(-|\boldsymbol{a} - \boldsymbol{a}^*|/R_c). \tag{17}$$

Obviously the 'localisation' near the pattern $\{\xi^{(\alpha_0)}\}$ $(\langle|\boldsymbol{a}-\boldsymbol{a}^*|\rangle \sim R_c)$ will take place if

$$\tfrac{1}{2}N\beta^3 h_0^2 J_0 q_{\alpha_0}^2 \gtrsim \ln(N/R_c^2). \tag{18}$$

In other words, if

$$q_{\alpha_0}^2 \gtrsim q_c^2 \sim \frac{2}{\beta^3 h_0^2 y_0} \frac{\ln(N/R_c^2)}{N} \tag{19}$$

the pattern $\{\sigma^{(0)}\}$ 'localises' eventually near the pattern $\{\xi^{(\alpha_0)}\}$.

Of course, the true minimum of the Hamiltonian (5) is the pattern $\{\xi^{(\alpha_0)}\}$ and not the $\{\sigma^{(0)}(\boldsymbol{r}+\boldsymbol{a}^*)\}$ one. The retrieval of the pattern $\{\xi^{(\alpha_0)}\}$ will be due to the dynamical evolution of the σ. This second stage will start when $M^{(\alpha_0)}(\boldsymbol{a})$ reaches the value of h_0/J_0. The function of the model at this stage corresponds to the ordinary Hopfield model in which the initial pattern $\sigma^{(0)}(\boldsymbol{r}+\boldsymbol{a}^*)$ has a finite overlap with that in one of the stored memories.

There is a danger, however, that, due to a small value of the critical overlap q_c (equation (19)), the 'trajectory' of $\sigma^{(0)}(\boldsymbol{a})$ could appear in a 'region of attraction' of some other learnt pattern. Let us calculate the probability of such an event.

The number of states which have an overlap $\gtrsim q$ $(q \ll 1)$ with some given state is

$$n(q) \sim 2^N \exp(-\tfrac{1}{2}Nq^2). \tag{20}$$

Therefore the probability that $\{\sigma^{(0)}(\boldsymbol{r}+\boldsymbol{a})\}$ will appear (with the overlap q) near one of the p stored uncorrelated patterns is

$$P(q) \sim p(N/R_c^2) \exp(-\tfrac{1}{2}Nq^2). \tag{21}$$

For $p = \alpha N$ the probability of being 'caught up' by one of the 'wrong' patterns is

$$P(q_c) \sim \alpha(N^2/R_c^2) \exp(-\tfrac{1}{2}Nq_c^2) = \alpha R_c^2 (N/R_c^2)^{2-T^3/J_0 h_0^2}. \tag{22}$$

It means that the system will function smoothly at the temperatures

$$T > T_c = J_0(2h^2/J_0^2)^{1/3} \tag{23}$$

The restriction $T < J_0$ and $\alpha < \alpha_c \simeq 0.138$ (Amit *et al* 1985) valid for the Hopfield model hold here too.

The generalisation of the above considerations for rotations and scaling is straightforward. The fields $\varepsilon(\boldsymbol{r})$ of the Hamiltonian (5) should be defined as follows:

$$\varepsilon(\boldsymbol{r}; t) = h_0 \sigma^{(0)}([\lambda(t)\hat{\theta}(t)\boldsymbol{r}] + \boldsymbol{a}(t)). \tag{24}$$

The effective potential for the variables λ, θ and $\boldsymbol{a}$ will have a form:

$$E(\lambda, \theta, \boldsymbol{a}) \simeq -\tfrac{1}{2}N\beta^2 h_0^2 J_0 \sum_{\alpha=1}^{p} (M_0^{(\alpha)}(\lambda, \theta, \boldsymbol{a}))^2 \tag{25}$$

where

$$M_0^{(\alpha)}(\lambda, \theta, \boldsymbol{a}) = \frac{1}{N} \sum_{\boldsymbol{r}} \xi^{(\alpha)}(\boldsymbol{r}) \sigma^{(0)}([\lambda\hat{\theta}\boldsymbol{r}] + \boldsymbol{a}). \tag{26}$$

The overlap with the pattern $\{\xi^{(\alpha_0)}\}$ near the optimal values of λ^*, θ^* and $\boldsymbol{a}^*$ could be represented as

$$M^{(\alpha_0)}(\lambda, \theta, \boldsymbol{a}) \simeq q_{\alpha_0} \exp(-|\lambda-\lambda^*|/\lambda_c) \exp(-|\theta-\theta^*|/\theta_c) \exp(-|\boldsymbol{a}-\boldsymbol{a}^*|/R_c) \tag{27}$$

where

$$\theta_c \sim R_c/\sqrt{N} \qquad \lambda_c \sim R_c\sqrt{N} \tag{28}$$

are the 'correlation angle' and the 'correlation scale', respectively.

Therefore the critical overlap q_c needed for the 'localisation' near the pattern $\{\xi^{(\alpha_0)}\}$ $(\langle|\theta-\theta^*|\rangle \sim \theta_c, \langle|\lambda-\lambda^*|\rangle \sim \lambda_c)$ remains the same as in (19). Consequently the model considered will exhibit good retrieval in the temperature interval $T_c < T < J_0$, where T_c is given by (23).

Obviously the model discussed in this letter could be easily modified to include a biologically motivated asymmetry of interactions, dilution and so on. Its present (very simple) form could be considered as a demonstration of a rather promising idea of using neural thresholds as additional degrees of freedom. The model seems rather convenient for computer tests.

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References

Amit D J, Gutfreund H and Sompolinsky H 1985 *Phys. Rev. Lett.* **55** 1530
Derrida B, Gardner E and Zippelius A 1987 *Europhys. Lett.* **4** 167
Domany E, Meir R and Kinzel W 1986 *Europhys. Lett.* **2** 175
Dotsenko V S 1985 *J. Phys. C: Solid State Phys.* **18** L1017
—— 1986 *Physica* **140A** 410
Feigelman M V and Ioffe L B 1987 *Int. J. Mod. Phys.* B **1** 51
Gutfreund H 1987 *Preprint*
Hertz J A, Grinstein G and Solla S A 1986 *Preprint* Nordita 86/18
Hopfield J J 1982 *Proc. Natl Acad. Sci. USA* **79** 2554
—— 1984 *Proc. Natl Acad. Sci. USA* **81** 3088
Hopfield J J, Feinstein D I and Palmer R G 1983 *Nature* **304** 158
Kree R and Zippelius A 1988 *J. Phys. A: Math. Gen.* **21** to be published
Little W A 1974 *Math. Biosci.* **19** 101
Parga N and Virasoro M 1986 *The ultrametric organisation of memories in neural networks. ICTP preprint*
Shinomoto S 1987 *Biol. Cybern.* **57** 197
Sompolinsky H and Kanter I 1986 *Phys. Rev. Lett.* **57** 2861
Treves A and Amit D J 1988 *J. Phys. A: Math. Gen.* **21** 3155
von der Malsburg C and Bienenstock E 1987 *Europhys. Lett.* **3** 1243